CR-124,043

AN EXPERIMENTAL INVESTIGATION OF THE LAMINAR BOUNDARY LAYER ON A ROTATING FLAT PLATE

by

John Bevill Peterson, Jr.

Thesis submitted to the Faculty of the University of Maryland
in partial fulfillment of the requirements of the degree of
Master of Science
1958

N71 72543
(ACCESSION NUMBER)
47
(PAGES)
TMX-67043
(NASA CR OR TMX OR AD NUMBER)
(THRU)
None
(CODE)
(CATEGORY)
FACILITY FORM 602

APPROVAL SHEET

Title of Thesis: An Experimental Investigation of the Laminar Boundary Layer on a Rotating Flat Plate

Name of Candidate: John B. Peterson, Jr.
Master of Science, 1958

Thesis and Abstract Approved: _____

Francis R. Hama
Professor
Institute for Fluid Dynamics

Date approved: _____

ABSTRACT

Title of Thesis: An Experimental Investigation of the Laminar Boundary Layer on a Rotating Flat Plate

John Bevill Peterson, Jr., Master of Science, 1958

Thesis directed by: Dr. Francis R. Hama

The three-dimensional laminar boundary layer on a rotating flat plate was measured experimentally and the results are compared with published theoretical solutions. The layer velocity profiles were measured at three chord-wise stations at the midspan of a 4-foot span rotating flat plate with a 1-1/2 foot chord. Experimental results near the leading edge agree well with theory, but significant variations from theory are found at large distances from the leading edge. Also given is a more accurate method of calibrating a hot-wire anemometer probe for measuring the speed and direction of the flow in a three-dimensional boundary layer.

VITA

Name: John Bevill Peterson, Jr.

Permanent address: 7818 Overhill Rd., Bethesda, Maryland.

Degree and date to be conferred: M. S., 1958.

Date of birth: October 24, 1930.

Secondary education: Bethesda - Chevy Chase High School, 1948.

Collegiat institutions attended	Dates	Degree	Date of Degree
University of New Mexico	1948-52	B.S.	1952
University of Maryland	1955-57	M.S.	1958

Major: Aeronautical Engineering.

Minor: Mathematics.

Positions held: Research Scientist at National Advisory Committee for Aeronautics, Langley Field, Virginia.

AN EXPERIMENTAL INVESTIGATION OF THE LAMINAR BOUNDARY LAYER ON A ROTATING FLAT PLATE

by
John Bevill Peterson, Jr.

Thesis submitted to the faculty of the University of Maryland
in partial fulfillment of the requirements of the degree of
Master of Science
1958

ACKNOWLEDGEMENTS

The author wishes to acknowledge the guidance and assistance of Dr. Francis R. Hama of the Institute for Fluid Dynamics and Applied Mathematics, of the University of Maryland. The author is also indebted to Dr. S. F. Shen of the Aeronautical Engineering Depart of the University of Maryland for suggesting the experiment.

TABLE OF CONTENTS

Chapter		Page
ACKNOWLEDGEMENT		ii
I.	INTRODUCTION	1
II.	PROCEEDURE	3
III.	DESCRIPTION OF APPARATUS	6
	A. Rotating Blade	6
	B. Hot-Wire Bridge	6
	C. Hot-Wire Anemometer Probe	7
	D. Traversing Apparatus	8
IV.	TEST AND RESULTS	9
V.	CONCLUSIONS	11
APPENDIX A.	SYMBOLS	12
APPENDIX B.	CALIBRATION OF THE V-WIRE HOT-WIRE ANEMOMETER.	14
APPENDIX C.	EXPERIMENTAL ACCURACY	21
SELECTED BIBLIOGRAPHY		26

LIST OF FIGURES

Figure		Page
1.	Rotating Flat Plate Apparatus	27
2.	Diagram of Stations on Blade	28
3.	Brush and Slip Ring Set	29
4.	Hot-Wire Probe and Traversing Apparatus	29
5a.	Nondimensional Boundary Layer Profiles at $x = 3.75$ Inches and $y = 21.5$ Inches	30
5b.	Nondimensional Boundary Layer Profiles at $x = 8.25$ Inches and $y = 21.5$ Inches	31
5c.	Nondimensional Boundary Layer Profiles at $x = 12.75$ Inches and $y = 21.5$ Inches	32
6.	Angles on the V-Wire Hot-Wire Anemometer Probe	33
7a.	Calibration of V-Wire Hot-Wire Anemometer	34
7b.	Values of r Obtained for V-Wire Hot-Wire	35
7c.	A_1 and A_2 Obtained for V-Wire Hot-Wire Anemometer. .	36
7d.	Variation of Heating Current With Angle at 25 ft/sec. .	37
7e.	Determination of θ for V-Wire Hot-Wire Anemometer . .	38
7f.	Summary Plot of Calibrations of V-Wire Hot-Wire Anemom Anemometer	39

CHAPTER I

INTRODUCTION

The wide spread use of helicopters, turbine jets, and more recently, VTOL aircraft has made the study of the three dimensional boundary layer on rotating airfoil sections one of practical importance. For this reason an investigation of the boundary layer on a rotating flat plate was made in the aeronautical engineering laboratory at the University of Maryland as a first step toward an understanding of three-dimensional boundary layers on rotating airfoils. The centrifugal acceleration and Coriolis force acting on the air cause the boundary layer over rotating blades to become three dimensional. Therefore calculations of the boundary layer by solving the Navier-Stokes equations in three dimensions becomes very complicated and approximate methods must be employed. Fogarty[1] and Liu[2] have calculated the three-dimensional boundary layer on a rotating flat plate, but very little experimental evidence has been published to support the mathematically calculated results.

The results presented in this experiment are the measurements of the boundary layer on a rotating flat plate 8 feet long and with a

---

[1] Fogarty, Laurence Eugene, The Laminar Boundary Layer on a Rotating Blade, Journal Aero. Science, vol 18, no. 4, April, 1951, pp 247-252.

[2] Liu, Shao-Wen, The Laminar Boundary Layer Flow on a Rotating Cylinder, Part I, Flat Plate, Cornell University Graduate School of Engineering.

1-1/2 foot chord, pivoted at the center chord point. Measurements were made approximately 2 feet from the center of rotations (the semispan position) and at rotational speeds of from 21.5 radians per second to 26.5 radians per second.

CHAPTER II

PROCEEDURE

The problem of measuring the boundary layer on a rotating flat plate is mainly one of transferring information obtained by a measuring instrument on the rotating blade to equipment and the operator stationary on the ground. Also it must be possible to traverse the instrument across the boundary layer while the rotating blade is in motion unless a large number of instruments are placed at various heights.

For this experiment the V-wire hot-wire anemometer was used as the sensing instrument. Even though there was no need for rapid response to velocity variations as is usually the case where the hot-wire anemometer is used, for the following reasons it was felt that the V-wire hot-wire anemometer was the simplest way to obtain the information needed in the limited number of tests run.

The most reliable and commonly used instrument to measure the boundary layer on stationary bodies is the total head tube. However, when the direction of flow must also be found the usual practice is to use three total head tubes placed at an angle to each other and then rotate the instrument until it is aligned with the flow as indicated by equal pressures on the two tubes angled to both sides of the center tube. The angle of rotation of the instrument as a whole is then an indication of the flow angle. Many three-dimensional boundary layers such as that on a swept wing have been measured in this way. The use of such an instrument on the rotating flat plate would require a mechanism to both

traverse the boundary layer and rotate the probe. Also a method of remotely indicating the position of the probe both in height and rotation angle would be necessary. The simplest method of doing this would probably be electrically which would require slip rings on the rotating shaft. Construction of a mechanism sensitive enough to obtain the required accuracies and rugged enough to withstand the high centrifugal forces (about 1000 ft/sec^2) on the blade without disturbing this accuracy would be very complicated. Also some method of measuring the pressures on the total head tubs is necessary. These pressures could possibly be read from a manometer as done by Persh and Sherwood[3] or on electrical pressure cell mounted in the rotating shaft but the centrifugal forces will greatly reduce the accuracy. Another and probably more satisfactory method would be to lead the pressures through a rotating pressure seal to a sensitive monometer stationary on the ground.

The requirement of rotating the probe in order to measure angle of flow greatly complicates the problem of constructing an appratus rugged enough to withstand the high centrifugal forces, therefore a probe that could sense angle of flow without being rotated was used. Although a pressure probe could be constructed and calibrated to obtain angle of flow information without rotation, a V-wire Hot-wire anemometer was used since the pressure probe could require the more complicated rotating pressure seals.

---

[3] Persh, Jerome and Sherwood, A. Wiley, An Experimental Investigation of the Boundary-Layer Flow on a Rotating Flat Plate, Journal Aero. Science, vol. 23, no. 7, July 1956, pp 703-704.

By careful calibration the V-wire hot-wire anemometer can be made as accurate as desired by simply using electrical instruments sensitive enough to detect the smallest change in bridge balance and hot-wire current necessary to give that accuracy.

CHAPTER III

DESCRIPTION OF APPARATUS

Rotating Blade

The rotating blade used in this experiment is the same as used by Persh and Sherwood[4]. A picture of the apparatus is shown in Figure 1. The blade is 8 feet long and 1-1/2 feet wide. It is made of aluminum, 1-1/2 inches thick and is beveled under the leading and trailing edge at 30° as shown in Figure 2(b). The blade is pivoted in the center and driven by a two horsepower direct current electric motor through a belt and pulley system at speeds up to about 50 radians per second. In this experiment a large trailing edge flap 1-1/2 feet wide and deflected at 15° was attached to the blade preceeding the test section of the blade in order to deflect the wake of the preceeding blade away from the test section. This flap combined with the small leading edge extension 2.375 inches wide and curved 0.3 inches below the plane of the top of the blade as shown in Figure 2 made the flow on the test section laminar. However, the large flap limited the speed of the blade to about 30 radians per second because of flutter at high speeds.

Hot-Wire Bridge

The hot-wire bridge used was a standard d. c. resistance bridge constructed for hot-wire anemometer work in the Institute of Fluid Dynamics of the University of Maryland. The hot-wire current was read

---

[4] Persh, Jerome and Sherwood, A. Wiley, op. cit.

on a Leeds and Northrup type K-2 potentiometer used in conjunction with a Leeds and Northrup standard 1 ohm resistor, a Leeds and Northrup standard cell, and a Minneapolis Honeywell electronic galvinometer. The hot-wire bridge balance was indicated by a Leeds and Northrup stabilized d. c. indicating microvoltmeter. Provision was made for switching either one of the two wires on the probe into the bridge circuit and a current reading was taken on each wire separately. The bridge voltage was taken from the same generator that supplied d. c. power to the driving motor for the blade.

The heating current for the hot wire and also the electrical power for the traversing apparatus was conducted through a set of slip rings and brushes to the rotating blade. A picture of the brush and ring set used in the experiment is shown in Figure 3. The brushes were made of phospher brass springs and the slip rings were brass. Except for rapid wearing of the rings no trouble was experienced and the voltage drop across the contacts was negligable at all rotational speeds.

Hot-Wire Anemometer Probe

A picture of the hot-wire probe used in the experiment is shown in Figure 4. The probe consisted of three sewing needles embedded in De Khotinsky cement which was coated with Dental Roc to prevent the needles from creeping under the high centrifugal forces. The hot-wires were 0.0004 inch diameter platinum wires which were soft soldered to the needles. Each wire was about 0.25 inches long and had a cold resistance of about 12 ohms.

The probe was calibrated in the aeronautical engineering laboratory at the University of Maryland using the same bridge and electrical instruments used in the experiment. The method used to calibrate the probe is describe in detail in Appendix B. The accuracy of the probe was $\pm$ 0.5° in angle of flow and $\pm$ 5 percent in velocity.

Traversing Apparatus

A picture of the traversing apparatus is shown in Figure 4. The hot-wire probe was mounted on a 0.75 inch by 5 inch plate which was pivoted at the back and raised and lowered in the front by a lead screw. The lead screw was rotated through a worm gear arrangement which was driven by an a. c. selsyn motor. The selsyn motor was connected to a selsyn transmitter on the ground through the slip ring and brush assembly. The height of the probe could be set at any height in the boundary layer with an accuracy of $\pm$ .0003 inches.

CHAPTER IV

TESTS AND RESULTS

All tests were made at 21.5 inches from the axis of rotation or approximately midway from the axis of rotation to the end of the blade. Measurements of the boundary layer at 3.75 inches, 8.25 inches, and 12.75 inches from the leading edge of the blade were made at two rotational speeds, approximately 22 radians per second and 26 radians per second. A diagram of the position of the stations used is shown in Figure 2. Measurements were also made with the 2.375 inch leading edge extension off, but the boundary layer profiles obtained were not consistent with the profiles measured with the extension on the blade, especially near the leading edge where some flow toward the axis of rotation was found. Evidently without the extension a small separation bubble formed at the leading edge of the blade which caused longitudinal vortices such as found in the boundary layer on a swept wing under some conditions.

Figures 5(a) through 5(c) show the experimental results obtained. For comparison the mathematical results obtained by Liu[5] for the boundary layer on a rotating flat plate with the axis of rotation displaced from the leading edge are shown as dashed lines. The nondimensional height, η, which was used by Liu is plotted against the velocity ratios in the boundary layer. Both the radial component of velocity, u_R, and the

---

[5] Liu, Shao-Wen, op. cit.

tangential component of velocity, u_θ, are normalized with the free-stream velocity, U_∞. The experimental and theoretical boundary layer profiles agree fairly well in $\frac{u_\theta}{U_\infty}$, but the agreement is very poor in $\frac{u_R}{U_\infty}$ except near the leading edge (Figure 5a).

As shown in Appendix C the overall estimated accuracy of the measured boundary layer profiles is better than $\pm .02$ in $\frac{u_R}{U_\infty}$ and about $\pm .05$ in $\frac{u_\theta}{U_\infty}$

CHAPTER V

CONCLUSIONS

The boundary layer velocity profiles obtained in this experiment support the theoretical results obtained by Liu[6] for stations near the leading edge. The calculations by Liu employ a series expansion in $\frac{x}{y}$ for $\frac{u_\theta}{U_\infty}$ and $\frac{u_R}{U_\infty}$, and the first and second order terms are calculated. As stated by Liu, the dropping of third order terms necessarily limits the region in which the solutions are expected to be valid, however the region in which $\frac{u_R}{U_\infty}$ is valid is evidently much less than the region where $\frac{u_\theta}{U_\infty}$ is valid. The reason for this is given in Liu's paper and he states that the radial component of velocity is expected to be reduced by the higher order terms.

---

[6] Liu, Shao-Wen, op. cit.

APPENDIX A

SYMBOLS

A	radiation and conduction heat loss term in hot-wire equation
B	coefficient of convection heat loss term in hot-wire equation
C', C"	constants
c_p	specific heat of air at constant pressure
d	diameter
h	coefficient of heat transfer
I	hot-wire current
k	thermal conductivity of air
l	length
Nu	Nusselt number, $\frac{hd_w}{k}$
Pr	Prandtl number, $\frac{\mu c_p}{k}$
r	the ratio $\frac{I_1^2-A_1}{I_2^2-A_2}$
R	hot-wire resistance
Re	Reynold's number, $\frac{\rho U d_w}{\mu}$
T	temperature, absolute
u	velocity in the boundary layer
U	velocity of air stream
x	distance from leading edge of blade measured along the chord
y	distance from axis of rotation measured along the span
z	distance normal to the surface of the blade

α resistivity coefficient of hot-wire

β angle between probe reference axis and air stream

γ angle between probe reference axis and arbitrary wind tunnel reference axis (Figure 6)

η nondimensional height in boundary layer, $\frac{z}{2}\sqrt{\frac{\Omega y}{\nu x}}$

θ half angle between hot-wires

μ viscosity of air

ν kinematic viscosity

ρ density of air

ψ angle between air stream axis and hot-wire (Figure 6)

Ω rotational speed of blade

Subscripts:

a hot-wire at ambient temperature

R in a radial direction

w heated hot-wire

θ in a tangential direction

1. left wire of hot-wire probe (Figure 6)

2. right wire of hot-wire probe (Figure 6)

∞ free-stream

APPENDIX B

CALIBRATION OF THE V-WIRE HOT-WIRE ANEMOMETER

Theory of the V-Wire Hot-Wire Anemometer

Since it was necessary to measure both the speed and directions of the air in the boundary layer on the rotating blade the V-wire hot-wire anemometer was used in this experiment. Whereas most V-wire hot-wire anemometers are rotated to be aligned to the flow and the angle of rotation is measured, the hot-wire in this experiment was not rotated but the current readings on each wire were compared in order to determine the flow angle. Although this method requires much more work to calibrate the wires and is probably not as accurate, the complicated apparatus necessary to rotate the wire and measure the angle of rotation while the plate to which the probe was attached is in motion was not available.

The equation for the heat loss of a hot wire as used in hot wire anemometer work is

$$Nu = (Pr \cdot Re)^{1/2} + Const. \tag{B1}$$

where

$$Nu = \frac{hd_w}{k} = \frac{I^2 R_w}{k\pi l_w (T_w - T_a)} \tag{B1a}$$

and

$$(Pr \cdot Re)^{1/2} = \left(\frac{\mu c_p}{k} \cdot \frac{\rho U d_w}{\mu}\right)^{1/2} \tag{B1b}$$

If the properties of the air remain constant then the equation for the hot wire can be written:

$$I^2 R_w = \left[C' + C''\sqrt{U}\right](T_w - T_a) \tag{B2}$$

But since

$$R_w - R_a = \alpha(T_w - T_a) \tag{B2a}$$

Then:

$$I^2 = \left[C' + C''\sqrt{U}\right]\frac{R_w - R_a}{\alpha R_w} \tag{B2b}$$

Since the overheat ratio, $\frac{R_w - R_a}{R_a}$, is fixed during the calibration, it is obvious that

$$I^2 = A + B\sqrt{U} \tag{B3}$$

Where A and B are found by calibration of the wire.

Previous investigations have shown that the hot-wire anemometer is sensitive only to the component of the air flow in the direction perpendicular to the hot wire (ie. U sin ψ, where ψ is the angle between the air direction and the hot wire as shown in Figure 6). Therefore equation (B3) can be written:

$$I^2 = A + B\,(U \sin \psi)^{1/2} \tag{B4}$$

This equation is accurate for flow angles (ψ) as low as 30°. Below 30° the accuracy of the equation is reduced.

During the operation of the V-wire hot-wire anemometer the direction and speed of the air flow are unknown. Only the current required to heat each hot wire is known. Using these currents the air speed perpendicular to each wire can be found. If the angles of the wires are known the direction and speed of the air flow can be found either graphically or by trigonometry. However, this method can be cumbersome and time consuming. Furthermore it is usually inaccurate since the physical angles of the wires is not necessarily the effective angles at which the wires act. Consequently, an alternate method which is outlined below was used.

Let the equations for wire 1 and wire 2 (see Figure 6) be:

$$I_1^2 = A_1 + B_1 (U \sin \psi_1)^{1/2} \tag{B5}$$

$$I_2^2 = A_2 + B_2 (U \sin \psi_2)^{1/2} \tag{B6}$$

where

$$\psi_1 = \theta_1 - \beta \tag{B7}$$

$$\psi_2 = \theta_2 + \beta \tag{B8}$$

U may be eliminated by dividing equation (B5) by equation (B6) which gives equation (B9), for $\theta_1 = \theta_2$

$$\left(\frac{B_2}{B_1}\right)\left(\frac{I_1^2 - A_1}{I_2^2 - A_2}\right) = \left[\frac{\sin(\theta - \beta)}{\sin(\theta + \beta)}\right]^{1/2} \qquad (B9)$$

Since the constants A_1, B_1, A_2, B_2, and θ are known from the probe calibration, the value of β corresponding to the measured currents can be found from this equation. After β is found, U is found from equation (B4) for either hot wire, using $\theta \pm \beta$ as the flow angle to the wire.

Calibration Proceedure

As stated in the above section, the constants A_1, B_1, A_2, B_2, and θ must be determined experimentally. Initially an attempt was made to determine the A's and B's by calibrating each of the hot wires in the probe independently. It was hoped that these constants, together with the geometric angle between the wires could be used in equation (B9) to determine the flow angle, β. But this proceedure was not satisfactory since the ratio $\frac{I_1^2 - A_1}{I_2^2 - A_2}$ appeared to be a function of velocity as well as the flow angle. This difficulty was overcome by determining effective values of the constants in such a manner that $\frac{I_1^2 - A_1}{I_2^2 - A_2}$ in a function of β only. The method of determining these effective constants will now be described in detail.

The probe was mounted in a wind tunnel as described in Appendix C. The air velocity was measured with a pitot static tube and the angle of

orientation of the probe (γ) was varied through 5 angles. This was done for 4 air speeds ranging from 15 feet per second to 35 feet per second. At each point the heating currents for the hot wires were measured. The square of the heating currents was then plotted against the square root of the velocities in the manner of Figure 7a (here only 3 of the γ are shown). From this plot an approximate A_1 (called A_1') is chosen. Using this trial A_1' and the measured currents, the ratio $\frac{I_1^2 - A_1'}{I_2^2 - A_2'}$ (called r) is calculated and plotted against trial A_1''s, (see Figure 7b). Notice that a group of lines is obtained for each γ. These lines correspond to different airspeeds. Since it is desired that r should be the same at a constant γ with varying velocity the A_2' at which r is most nearly constant is the value selected. Even though the A_1' and A_2' are not the final A 's to be used in equation (B9) can be shown that the r obtained in this way varies least with changes in velocity. Therefore the r's corresponding to the γ's are chosen from Figure 7b. The points chosen should lie on a straight line across the plot as shown by the dashed line. With these r's chosen, A_1 is plotted against A_2 as shown in Figure 7c for all the γ's and velocities used in the calibration. From this plot both the final A_1 and A_2 are chosen. In this case $A_1 = .00340$ and $A_2 = .00440$ were selected.

With A_1 and A_2 found the only problem remaining is to find the effective θ, the half angle between the hot wires (Figure 6) and consequently the ψ corresponding to each γ.

If the equation for the hot wire is taken as:

$$I_1^2 = A_1 + B_1 (U \sin \psi_1)^{1/2} \tag{B5}$$

Then since $\psi_1 = \text{const.} - \gamma$

$$\frac{d(I_1^2)}{d\gamma} = - \frac{B_1 (U)^{1/2}}{2 (\sin \psi_1)^{1/2}} \cos \psi_1 \tag{B10}$$

and

$$\tan \psi_1 = - \frac{I_1^2 - A_1}{2 \frac{d(I_1^2)}{d\gamma}} \tag{B11a}$$

Similarly

$$\tan \psi_2 = \frac{I_2^2 - A_2}{2 \frac{d(I_2^2)}{d\gamma}} \tag{B11b}$$

Then, for a constant velocity, I^2 is plotted against γ as shown in Figure 7d. The I^2 is found by interpolation from Figure 7a. The slopes, $\frac{dI^2}{d\gamma}$ are found from Figure 7d and used in equations 11a and 11b to obtain ψ_1 and ψ_2. The ψ's are then plotted against γ as shown in Figure 7e. The point at which the lines intersect, or where $\psi_1 = \psi_2$, is the point at which $\psi = \theta$, since $\psi_1 + \psi_2 = 2\theta$, (see Figure 6). The intersection of the curves of Figure 7e shows that $\theta = 44^\circ$.

With A_1, A_2, Θ and γ for $\beta = 0$ determined, a final summary plot as shown in Figure 7f can be constructed from the calibration data. With this graph the flow angle can readily be determined by measuring the heating currents in the V-wire hot-wire anemometer and calculating $\frac{I_1^2 - A_1}{I_2^2 - A_2}$

After the flow angle is known, the velocity of the flow can be found from either one of the wires by

$$U = \frac{(I^2 - A)^2}{B^2 \sin(\Theta \pm \beta)} \qquad (B12)$$

B is the slope of the curve of I^2 against $U^{1/2}$ as used in ordinary hot-wire work except that the intercept at $U = 0$ is kept at the A found during the flow angle calibration. Forcing the intercept in this way will result in some error in the velocity since the natural intercept as shown in Figure 7a is not the same. However, this error does not exceed a few percent and, in this experiment, it was much more important to obtain accurate angles than exact velocities.

APPENDIX C

EXPERIMENTAL ACCURACY

Calibration of Probe

The V-wire hot-wire anemometer was calibrated in the 2-1/2 foot wind tunnel at the aeronautical engineering laboratory of the University of Maryland. The probe was mounted in the center of the tunnel and rotated about an axis through the hot wires in order to avoid moving the probe into other regions of the flow in the tunnel where the air-stream angle may change. The velocity of the air stream in the tunnel was taken with a pitot-static tube and the pressure read on an electrical strain gage pressure pickup, which was calibrated against an alcohol manometer at a higher pressure immediately before and after each run. This allowed the velocity to be read accurate to $\pm$.5 feet per second in the range of velocities used in the calibration (15-35 feet per second).

Since the same hot-wire bridge and electrical instruments for measuring the wire current were used in the experiment as were used in the calibration, the spread of data used for the calibration was taken as the accuracy of the probe. This accuracy is conservative since it was found that the airflow in the boundary layer on the rotating blade was steadier than that in the wind tunnel which allowed the hot-wire bridge to be more accurately balanced during the experiment. As shown in Figure 7f the maximum error in angle is $\pm.5^{\circ}$ in the speed range of 15 to 35 feet per second. The speeds measured in the boundary layer varied approximately from 10 to 40 feet per second but the greater

speed range probably did not effect the accuracy of the probe. Calculating the velocity by using the constants found in the calibration, the maximum error was ±5 percent of the velocity as measured by the pitot-static tube. As explained in Appendix B, the large error in measuring the velocity is due to the fact that A_1 and A_2 are selected to give the best accuracy in the angle measurement and the velocity measurement suffers accordingly.

Expressed in terms of the quantities shown in the boundary layer profile graphs the accuracy is from $\pm.01 \frac{u_\theta}{U_\infty}$ to $\pm.015 \frac{u_\theta}{U_\infty}$ in $\frac{u_r}{U_\infty}$ and $\pm.05 \frac{u_\theta}{U_\infty}$ in $\frac{u_\theta}{U_\infty}$

Other Sources of Error

The probe must be placed in the boundary layer so that the exact position of the velocities measured by the probe are known. There was no way of measuring the position of the probe while the blade was rotating but for the following reasons it is believed the position did not change from the position set while the blade was stationary. The initial position of the probe was obtained when the hot-wire support needles touched the blade as determined by an ohmmeter between the probe and the blade. The initial position of the probe was set after the blade was brought up to speed and then checked after each set of measurements was taken in order to insure that the probe did not creep under the high centrifugal forces. No change in the initial position greater than .0006" was found in any test.

It is essential that the two hot wires of the probe be at the same height from the blade since a comparison of the velocities measured by each wire is used to obtain the flow angle and any variation in the heights of the two wires would give false readings. The wires were set as near as possible to the same height and careful measurements with a shop microscope and a vernier height gage showed that the maximum difference in height of the wires was .0007".

It can be shown that:

$$\frac{\Delta u_r}{U_\infty} \equiv \Delta z \frac{d\frac{u_\theta}{U_\infty}}{dz} \; 1/2(\cos \varepsilon \sin 2\Psi + \sin \varepsilon \sin \Psi) \qquad (C1)$$

Where Δz is the difference in the heights of the hot wires and ε is the flow angle with respect to the free-stream. This gives a maximum error of .01 in $\frac{u_R}{U_\infty}$.

The change in the heights of each wire was measured while the probe was under the centrifugal forces acting while the blade was rotating. A shim was placed under one of the three support needles in the probe. The change in the height of the needle from the static position was obtained by measuring the change in the initial position. In this way the difference in the heights of each of the three supports while rotating could be determined. The maximum change in Δz under the centrifugal force of the rotating blade was .0002" which can be considered negligable.

The accuracy of the probe would have been greatly effected by any change of resistance in the slip rings and brushes which carried the hot-wire heating current to the probe from the bridge on the ground. For this reason a precision resistor approximately the same value as the hot wire was substituted for the wire and the bridge current was adjusted to the same current used to heat the hot wire. After the bridge was balanced the blade was run up to the speed used in the experiment. No change was found in the bridge balance at any time, therefore any change in the resistance of the slip rings and brushes was negligable.

The combined error of all of the above sources is less than $\pm.02$ in $\frac{u_R}{U_\infty}$ and about $\pm.05$ in $\frac{u_\theta}{U_\infty}$.

When comparing the results with the theoretical boundary layer profiles from Liu[7], the following conditions of the experiment which were different from the conditions set in the theoritical solutions must be noted. In the experiment it was found that the air stream outside the boundary layer did not have the same speed as the physical speed of the blade in relation to the ground. The blade caused the air to swirl in the vicinity of the experiment causing a reduction in the relative velocity of the air over the blade. The swirl velocity was approximately 11 percent of the physical velocity of the blade.

In order to prevent separation of the flow at the leading edge it was found necessary to attach a leading edge extension to the blade as shown in Figure 2. However, the contour of the blade deviated only

---

[7] Liu, Shao-Wen, op. cit.

slightly from a flat plate and the effect of the drooped leading edge was probably negligable.

The solution given by Liu[8] is for a semi-infinite flat plate and some effect from the end of the blade tested might be expected. This effect is probably small, however since the tests were run at the midspan position, about 2 feet from the end of the blade.

---

[8] Liu, Shao-Wen, op. cit.

SELECTED BIBLIOGRAPHY

1. Fogarty, Laurence Eugene, The Laminar Boundary Layer on a Rotating Blade, Jour. Aero. Sci., vol. 18, no. 4, April 1951, pp 247-252.

2. Liu, Shao-Wen, The Laminar Boundary-Layer Flow on Rotating Cylinders, Part I, Flat Plate, Cornell University, Graduate School of Engineering.

3. Persh, Jerome and Sherwood, A. Wiley, An Experimental Investigation of the Boundary-Layer Flow on a Rotating Flat Plate, Jour. Aero. Sci., vol. 23, no. 7, July 1956, pp 703-704.

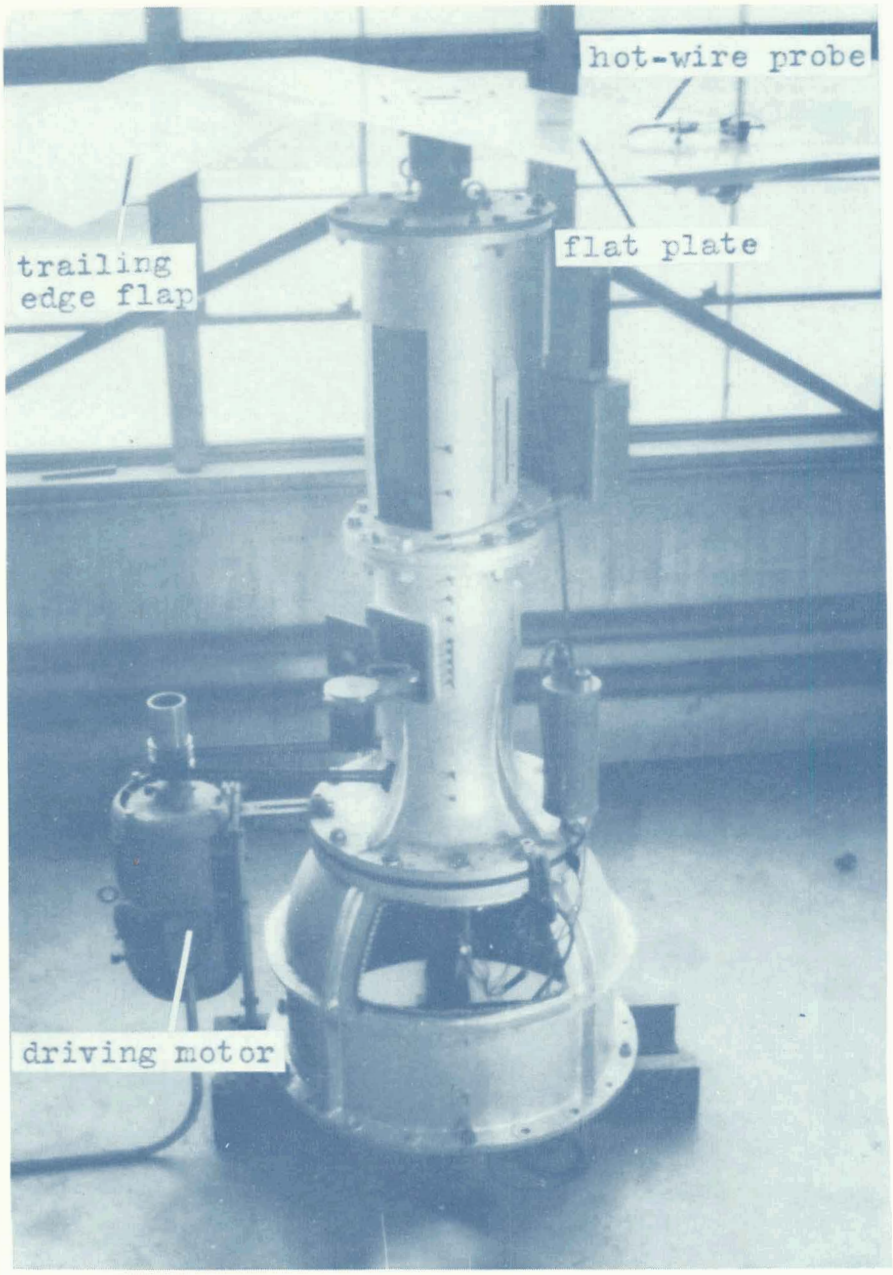


Figure 1.- Rotating flat plate apparatus. L-58-1669

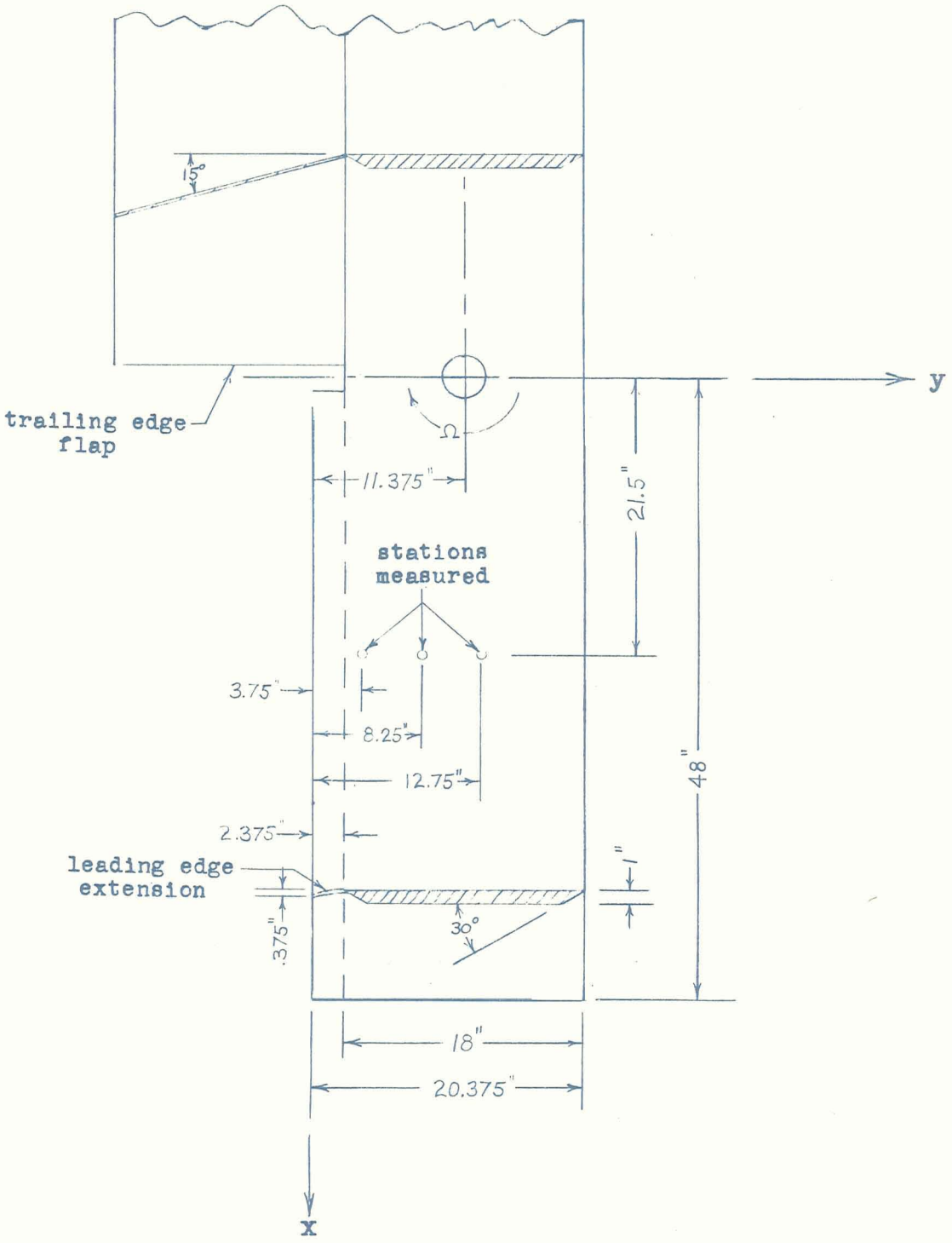


Figure 2.- Diagram of stations on blade.

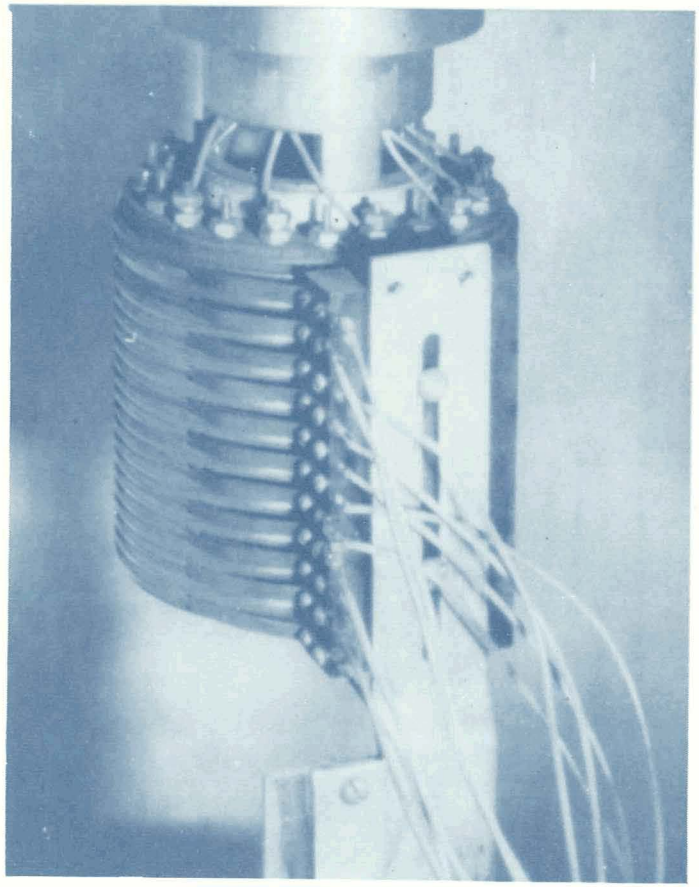
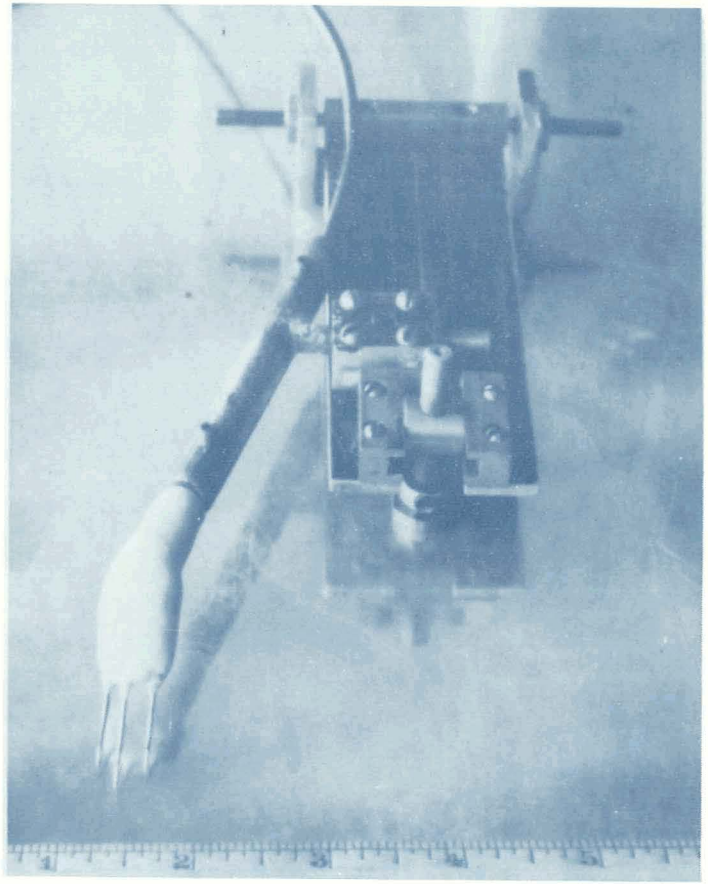


Figure 3.- Brush and slip ring set.



L-58-1670

Figure 4.- Hot-wire probe and traversing apparatus.

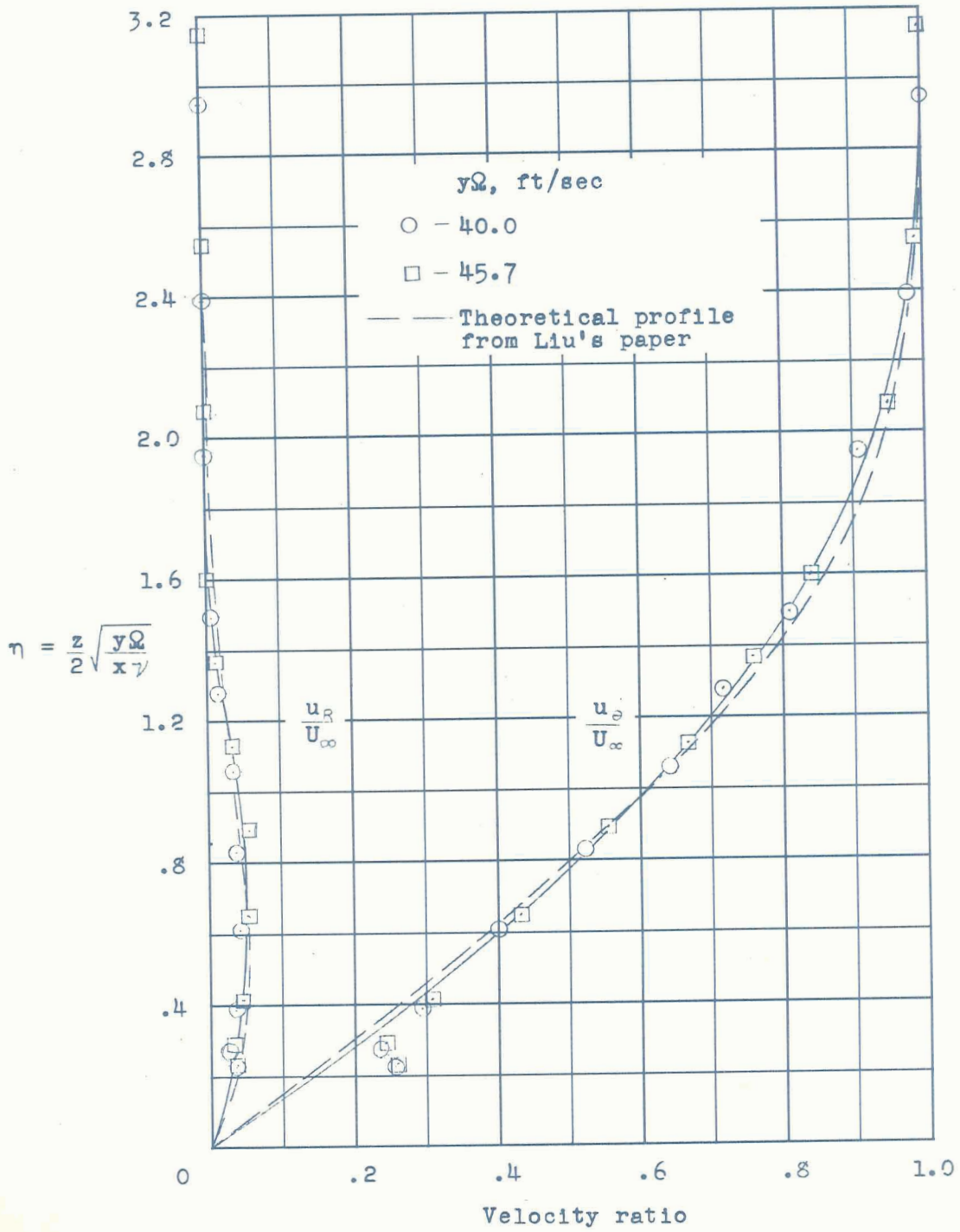


Figure 5a.- Nondimensional boundary-layer profiles at x = 3.75 in. and y = 21.5 in.

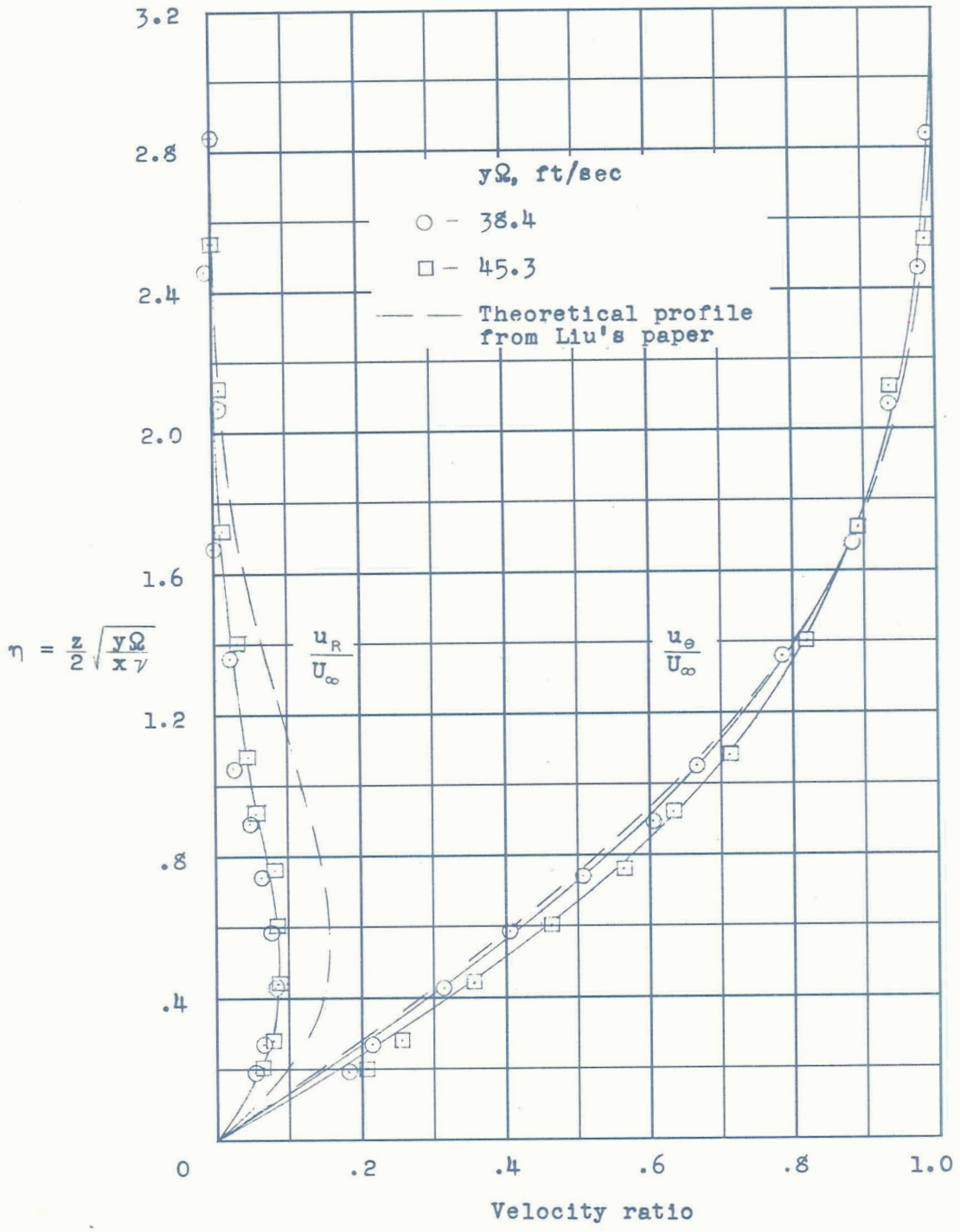


Figure 5b.- Nondimensional boundary-layer profiles at x = 8.25 in. and y = 21.5 in.

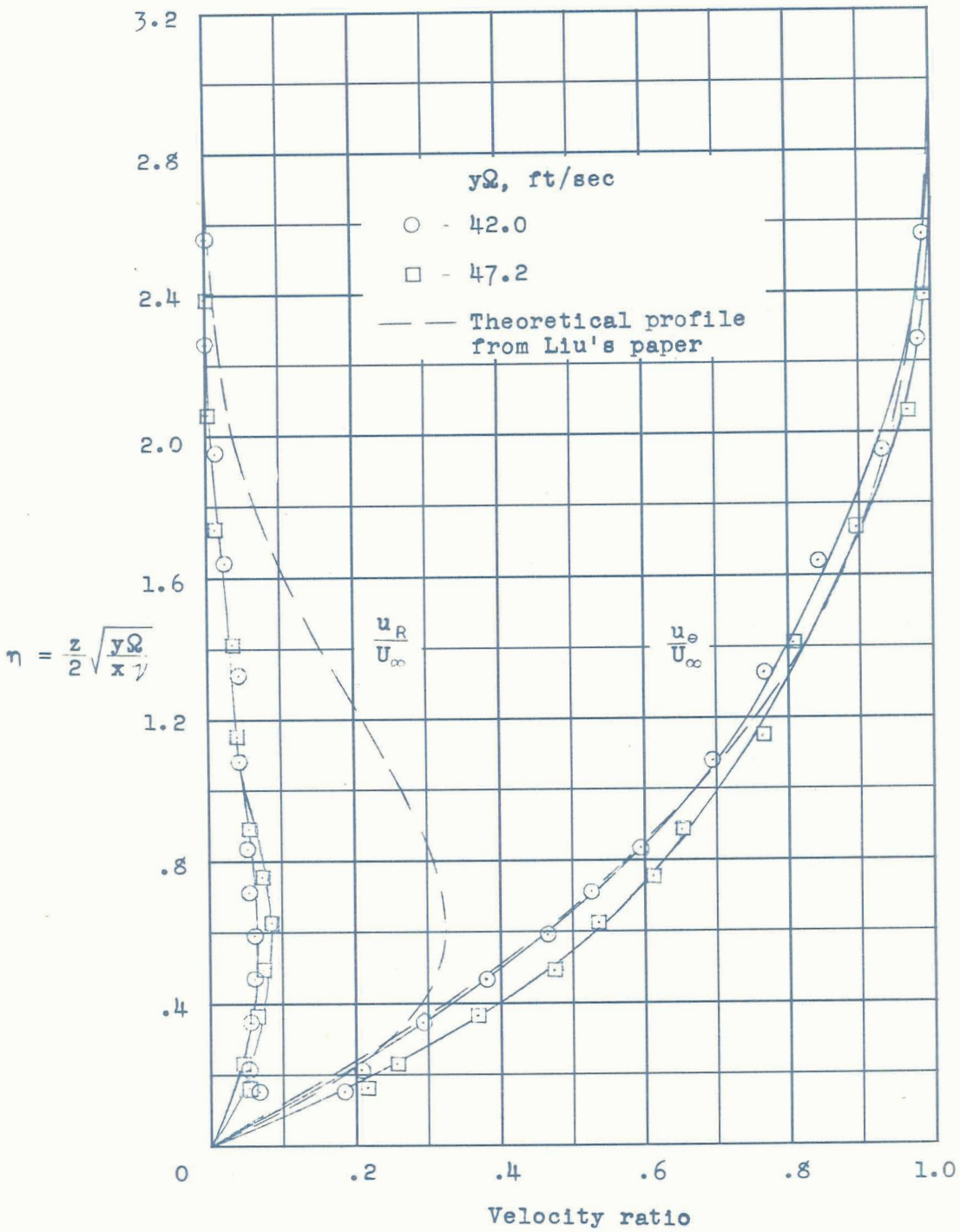


Figure 5c.- Nondimensional boundary-layer profiles at x = 12.75 in. and y = 21.5 in.

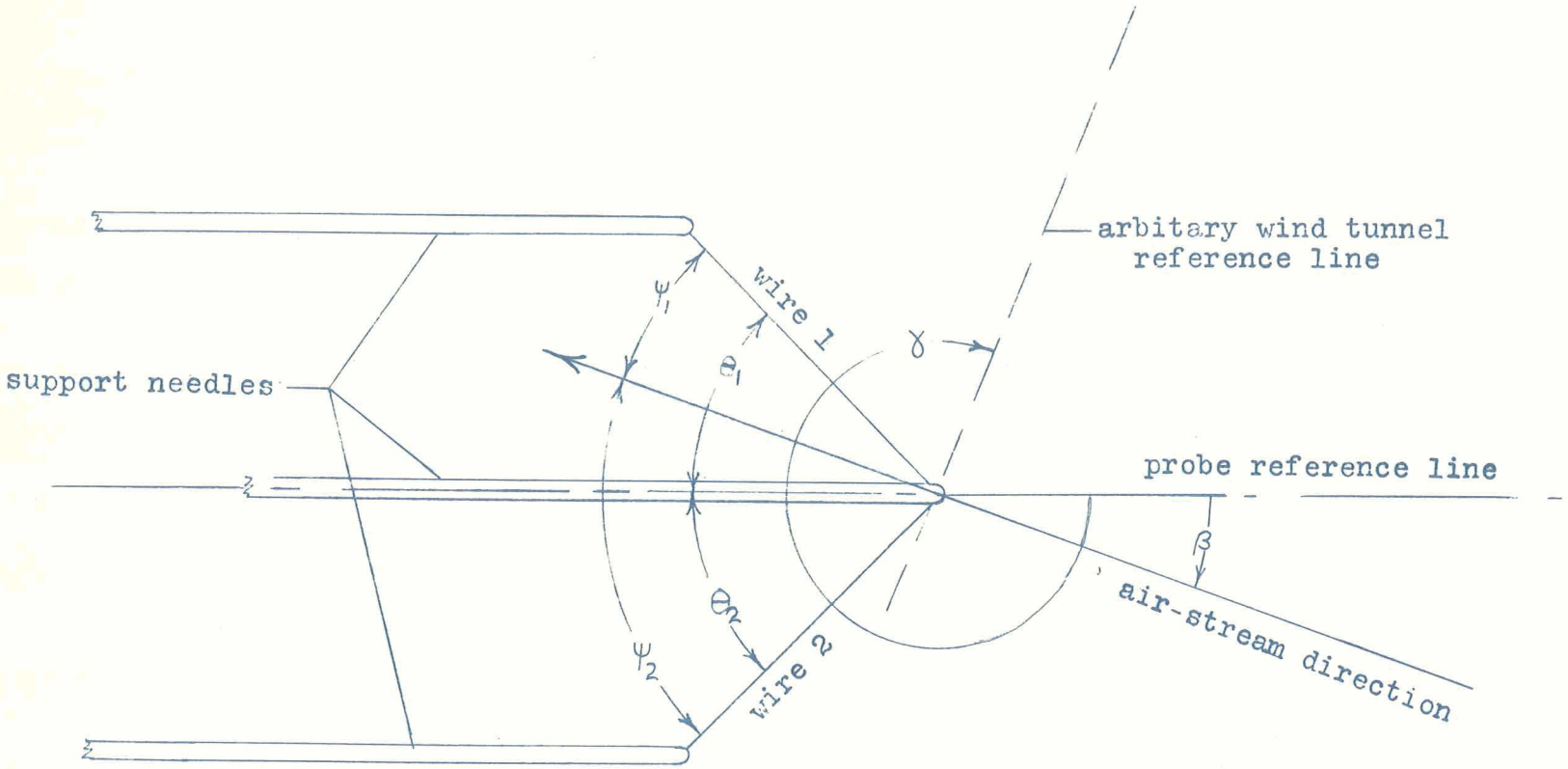


Figure 6.- Angles of the V-wire hot-wire anemometer probe.

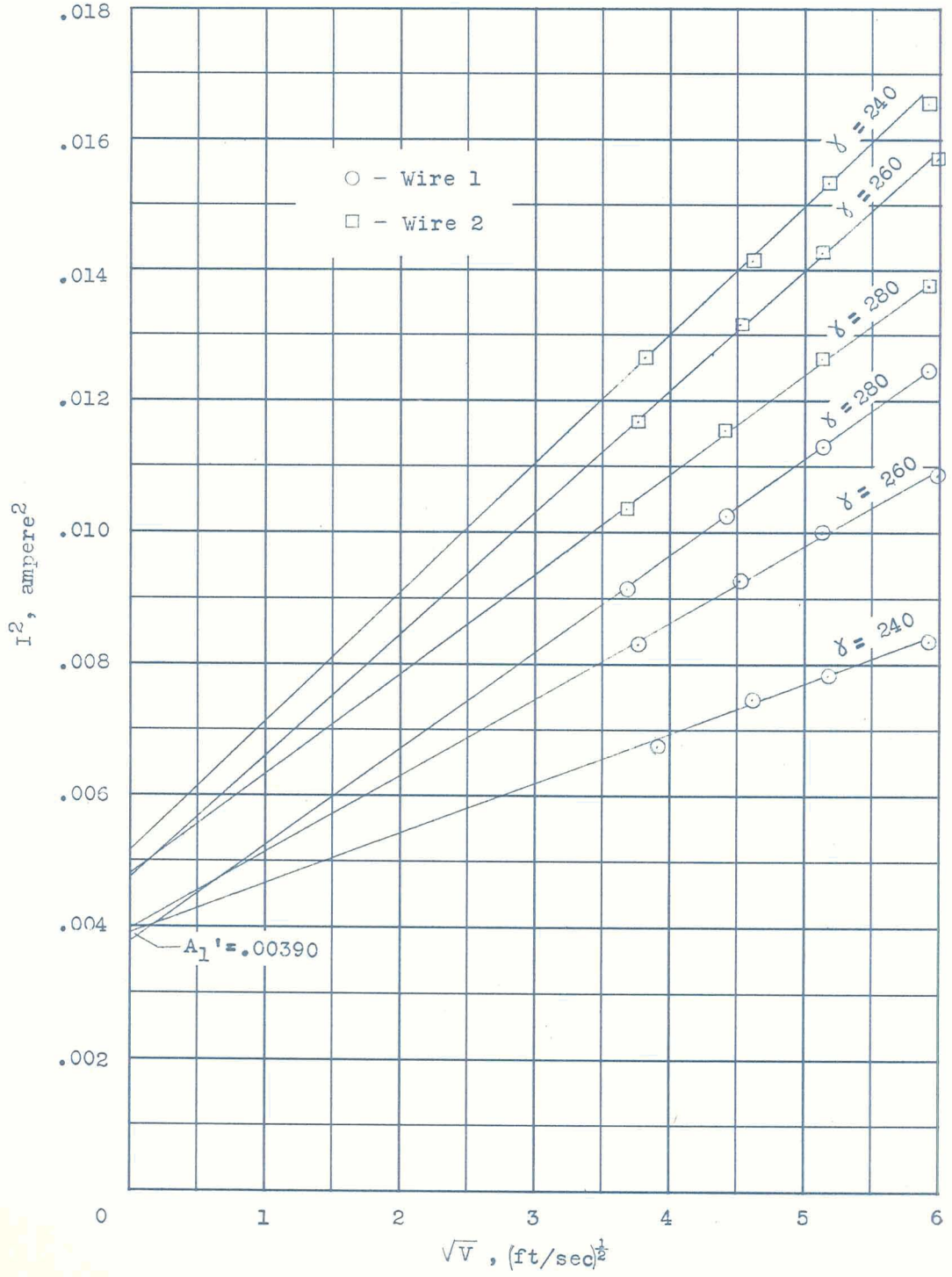


Figure 7a.- Calibration of V-wire hot-wire anemometer.

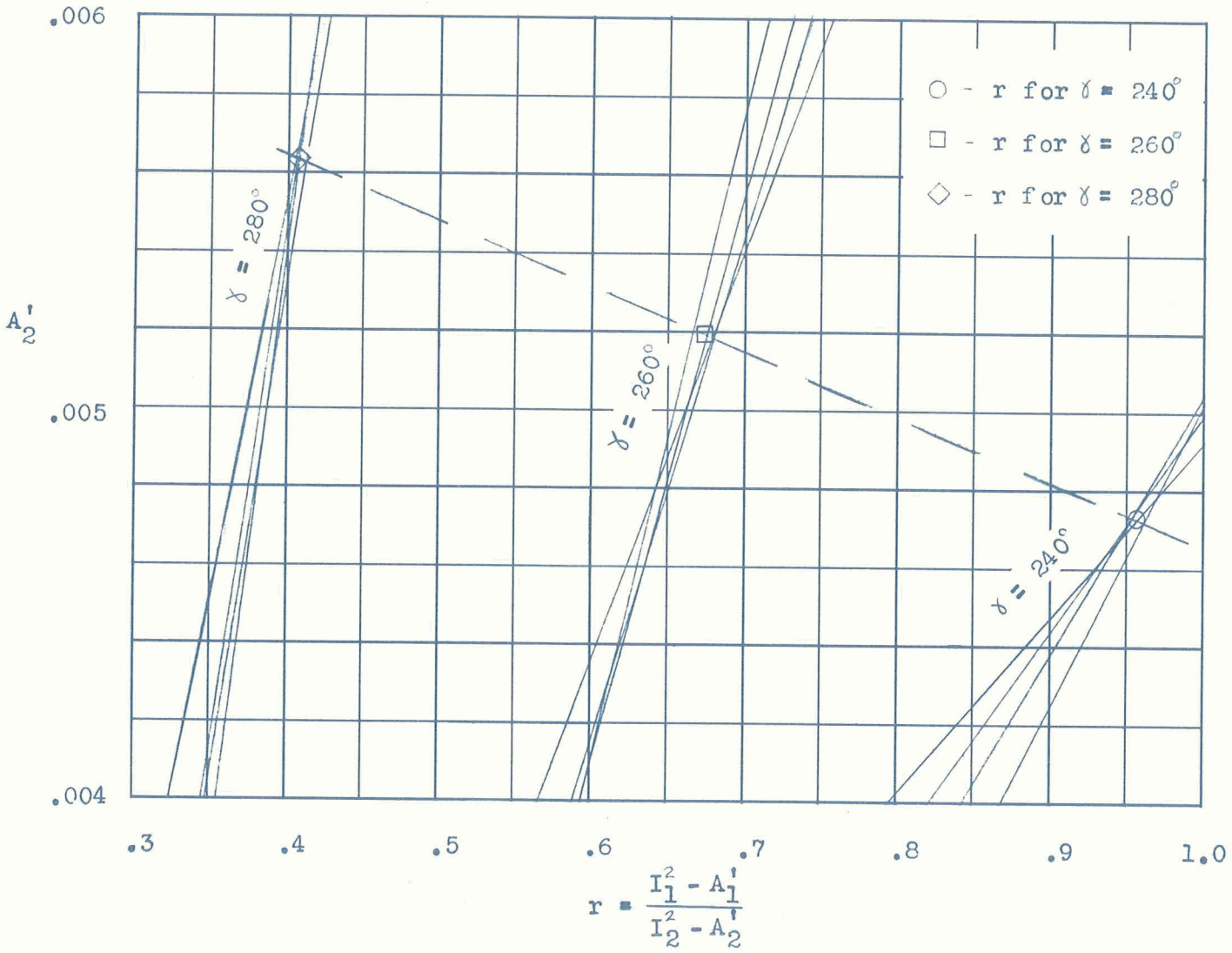


Figure 7b.- Values of r obtained for V-wire hot-wire anemometer.

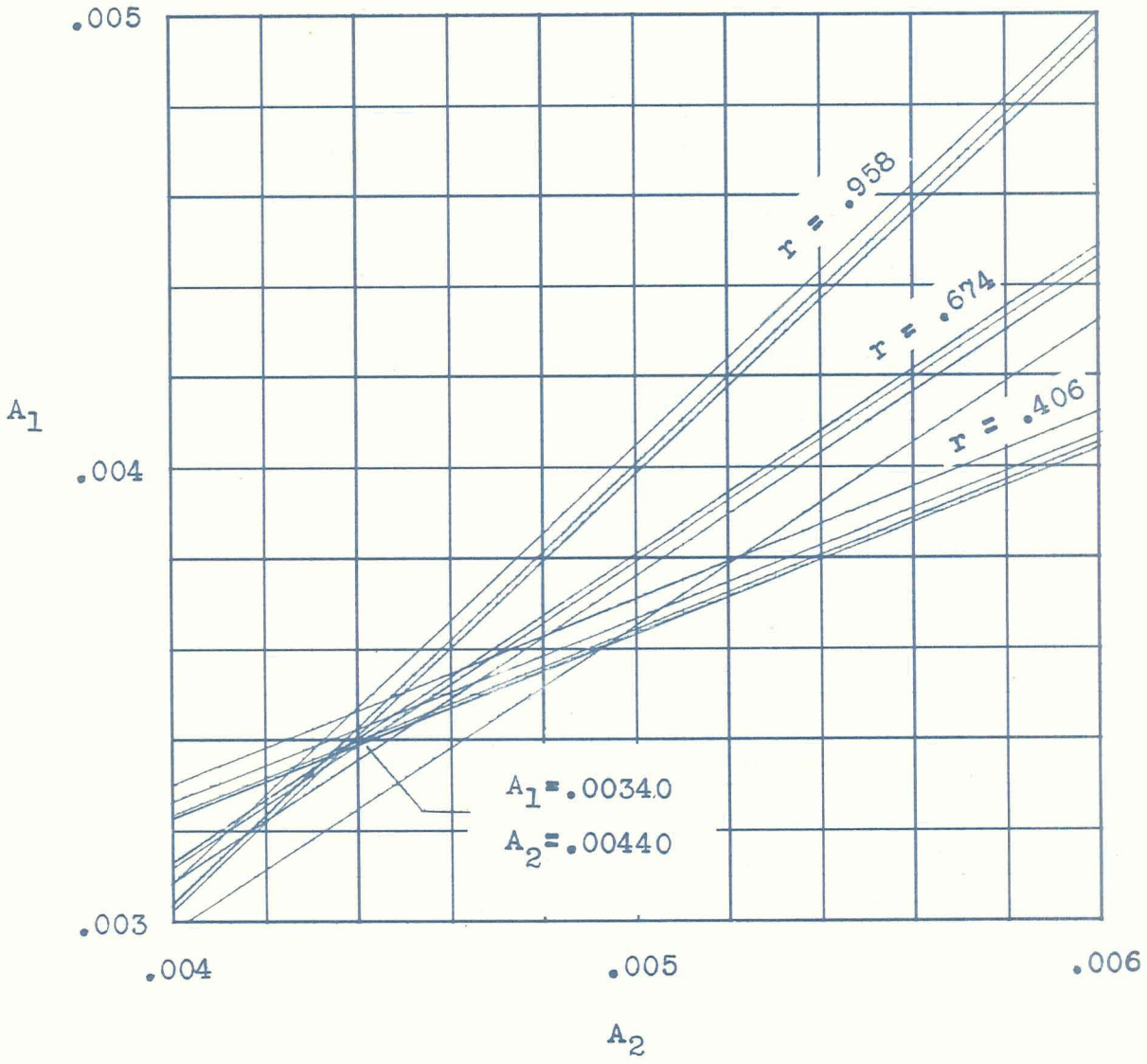


Figure 7c.- A_1 and A_2 obtained for V-wire hot-wire anemometer.

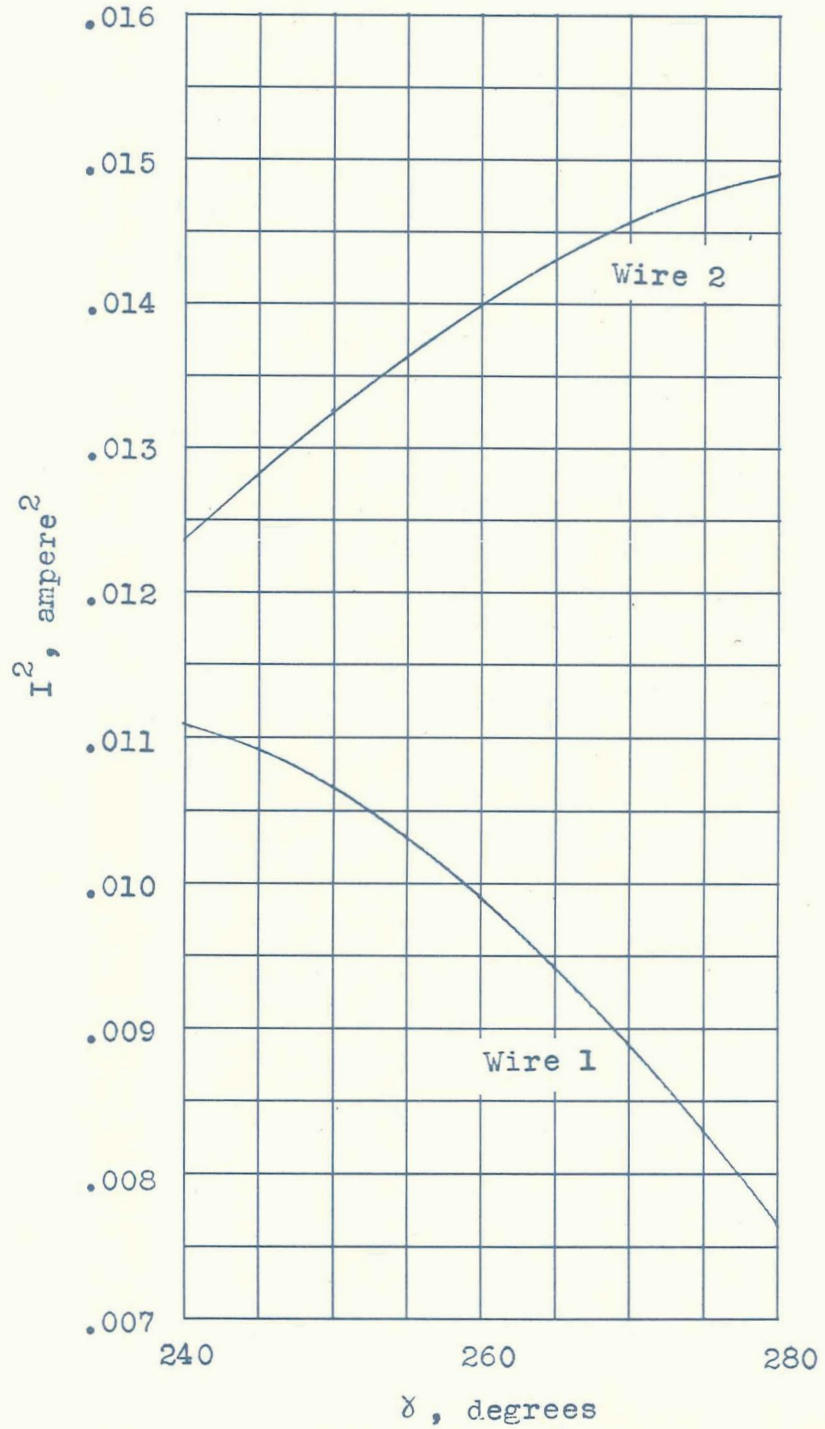


Figure 7d.- Variation of heating current with angle at 25 ft/sec.

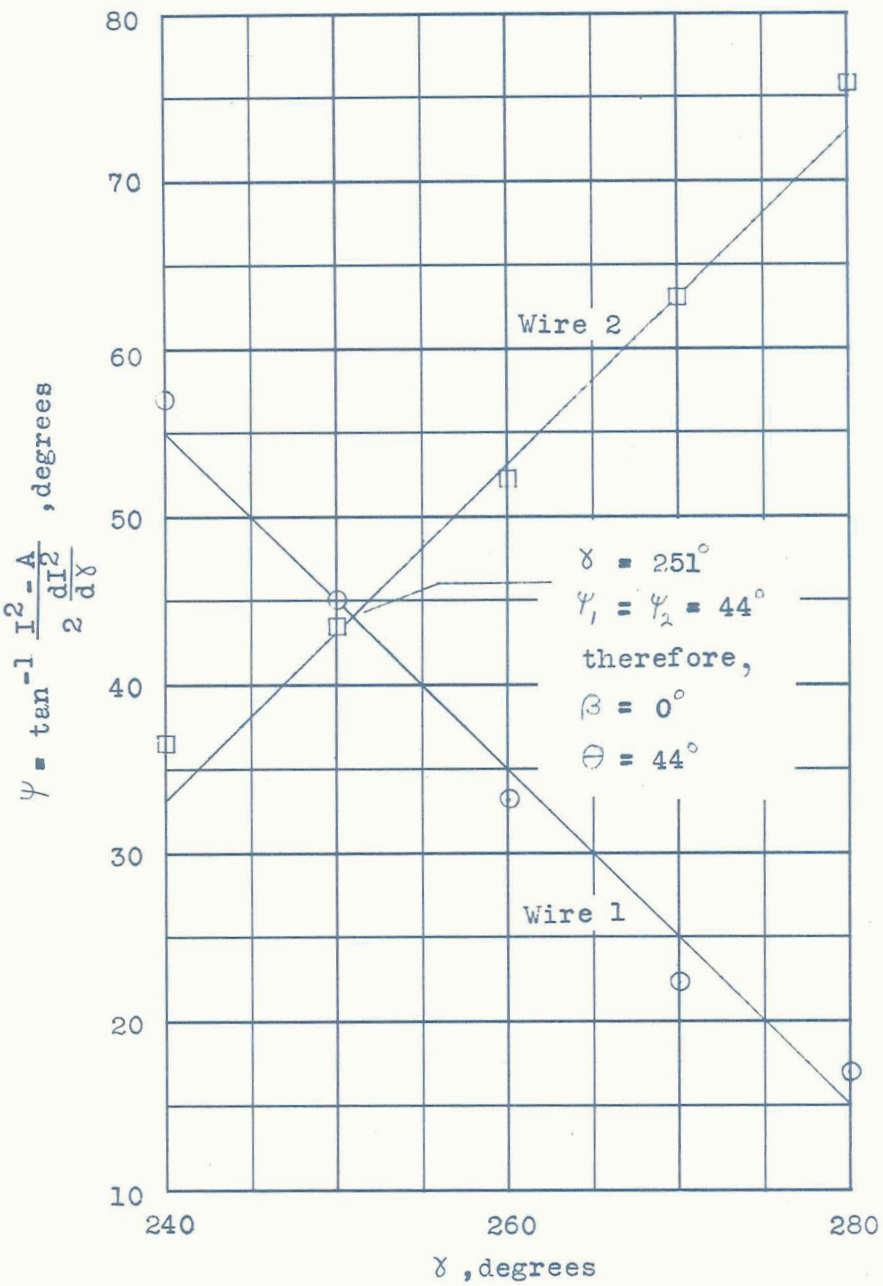


Figure 7e.- Determination of θ for V-wire hot-wire anemometer.

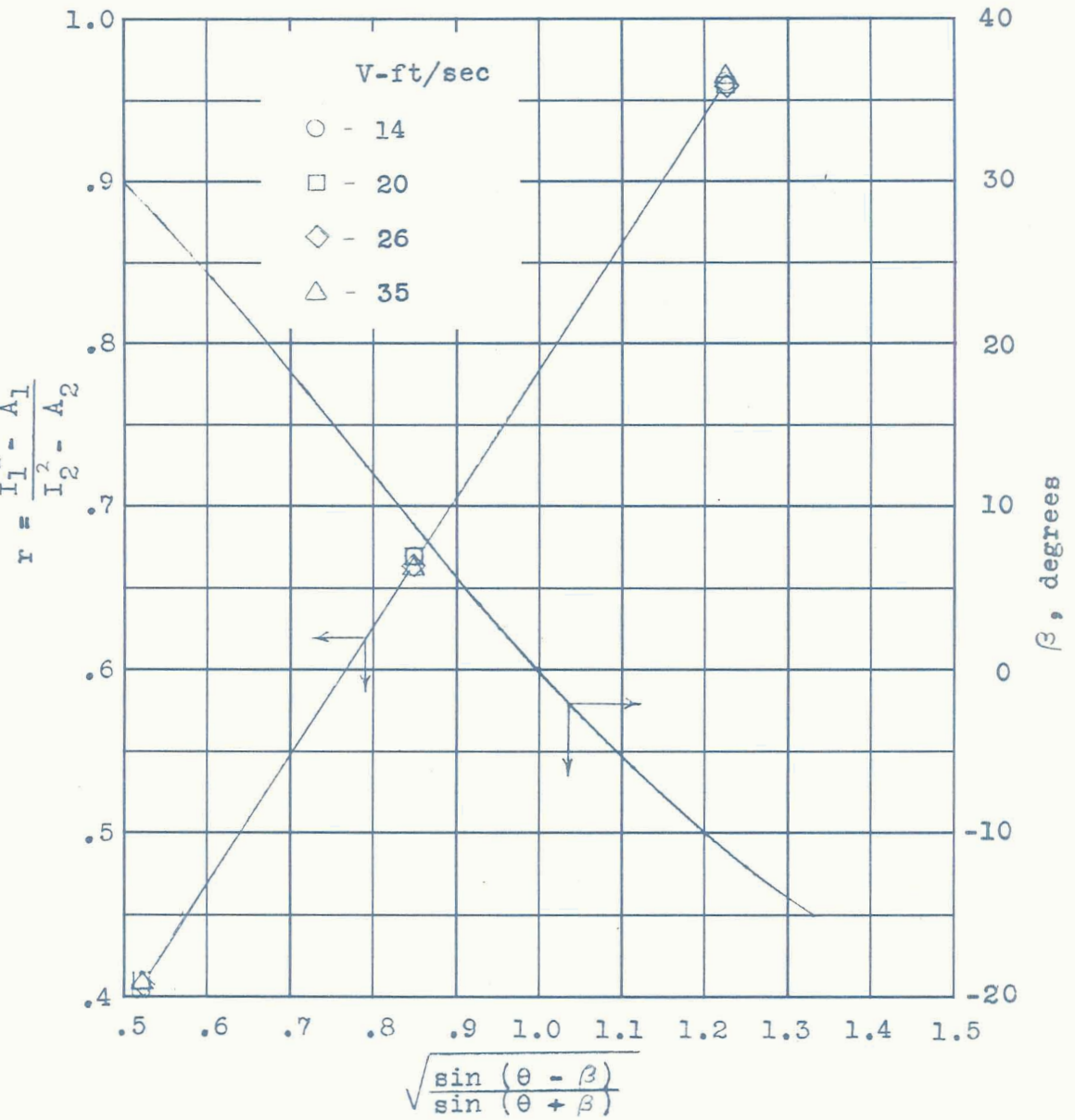


Figure 7f.- Summary plot of calibration of V-wire hot-wire anemometer.